**BBGKY Hierarchy**

So going back to the equation we derived in the classical Kubo formula file:



where **Ω** = {**r**i, **k**i} and i runs from 1 to N (treating p and k as synonyms, though **p** = ℏ**k**). Having the equation for the evolution of the full N-body distribution function, as was attempted in the Kubo approximation, is nice, but we’re often interested in a more restricted entity – the single particle distribution function. I guess this would suffice as long as the thing we’re interested in calculating is a single particle property – like average momentum, kinetic energy, etc. But if we wanted average energy, say, which might involve a two-particle potential, this would not be appropriate: we’d need at least the two particle distribution function.

**BBGKY Hierarchy for the single particle distribution function**

So anyway, we take this equation and integrate over N-1 particles.



where **Ω**1 = (**r**1, **k**1) of course and **F**ext is a single particle force, say coming from some external field, whether conservative or not, while **F**int is a two-particle interaction. Continuing,



where it’s been presumed that f is symmetric w/r to interchange of coordinates. This should be the case since the PDE is symmetric. Since the last integrand is odd with respect to interchange of **k**2, **k**3, it should go to zero. So I think we should just have:



Further, I believe the blue integrals are zero – perhaps because you can say the two particle distribution function must go to zero at the endpoints (I can see it going to zero for large k, but should it necessarily for large r – or really, r at the surface of the container? I guess so, at least since it cannot exit the container/substance ). So then we’d have (renaming Fext as simply F):



Although Hbath has been neglected, we will see that the effect of the other particles acting on our 1 in effect is a kind of thermal bath itself, as this is the term which re-establishes thermal equilibrium. So this equation finds use in calculating how the (single particle) distribution function evolves in response to external forces (coming from other particles). The RHS is problematic though, because it involves the two-particle distribution function. We could work out an equation for the f(**Ω**1,**Ω**2,t), but not surprisingly, it will involve the three-particle distribution function. We’d start with:



again. And integrate over N-2 particles,



And continuing,



The red integrals should be zero by IBP since the boundary term must go to zero one imagines. And the blue integral should go to zero because of oddness under exchange of **k**3,4. So we have:



and finally,



Basically the evolution of the n-particle distribution function involves both itself and the n+1 particle distribution. This is called the BBGKY hierarchy or something. And alas the equations do not naturally close on themselves – we have an infinite set.

**Specializing to a random single particle potential and two-particle potential**

So we’ll go back to the original equation:



We’ll be interested in two cases – presence of a random impurity potential, and presence of two particle interactions. The latter is already there on the RHS. The former is technically within the **F**ext [i.e., **F**] term, but we’ll make it more explicit. So let’s say these impurity potentials are randomly distributed throughout our substance at locations **R**1, **R**2, …, **R**Ni, where Ni is the number of impurities. Then we have (retaining possibility of some smooth external force **F**):



We do not know the positions of these forces, or at least we don’t want to have to know, and so we’d like to average over all impurity positions by applying (1/VN\_i)∫d3R1d3R2…d3RN\_i to both sides (it gives just 1 on the LHS, and 1 on the Fint term). Then we have:



And next we’ll discuss how we deal with this equation.